

METAPHORS AND METONYMIES USEFUL DEVICES OR NUISANCE? WHICH?

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What is a weed? A plant, whose virtue
has not yet been discovered.

Emerson

Everybody may admit that errors and mistakes are not desirable things, and who commits errors should not be happy about it. To the contrary, one is usually sorry in such circumstances and feels guilty. One sometimes feels one should be punished, and sometimes one even is punished. Who commits errors in school and brags about it, makes the unforgivable error, and deserves punishment. At last this is the traditional position of European pedagogy and is rarely questioned.

Of course some people talk about "blessed" errors but such talk is a sort of poor consolation for those who all too often commit such "blessed" errors. Pushing this sort of euphemistic phraseology a little further, some people talk about good errors and inevitably, by the law of action-and-reaction all the more stress the existence of haunting bad errors which are better to be avoided. There is always a grain of hypocrisy in such litotical euphemisms about errors. An error is a good error, only if it serves to expose another error which is often considered more serious than the first one.

In fact, every action taken is under the risk of making errors. There is no question about it, it is only a matter of magnitude of that risk. If the action taken is at someone's own initiative, then there is no question either who is responsible and who is to blame, and who should be ashamed of it.

This is in short the implicit phraseology of mistakes and errors, of crime, shame, and punishment. An error is no merit after all.

Is there no alternative for the point of view expressed implicitly by such error haunting phrases? They are all in place in well established authoritarian schools of pedagogy. They might be very damaging, in the circumstances where we place more value on creative learning. In spite of all efforts of many modern schools of pedagogy, the creative approach is not taken too seriously in school practice. In my country, the situation is even worse in this respect than the average. This is sad, because it is easy to discourage young people by improper words and small injustice, as perceived by them. It is then very difficult to repair the damage, especially in the case of those who are sensitive, shy, and very often talented.

What I would like to point out here is a sort of frame conflict very much like that described by Reddy (1979), Skemp (1978), Krygowska (1987) and many others on various occasions. If we really wish to encourage the creative learning, then we should pay much more attention to what the students actually mean and we should be less prone to insist on giving our own interpretation of what they literally say or write.

H. ers
In our joint paper, Bauersfeld and Zawadowski (1981), we pointed out that there exists in mathematics something analogous to figures of style in natural languages and that it is perhaps better to call these figures the figures of living thought. This is in complete agreement with the conclusions stated in Sadock (1979) and Black (1979). Such figures of living thought are especially conspicuous in living dialogues between students and teaching, and almost everywhere where mathematics, mathematical ideas and signs, are "in statu nascendi". This approach was used, e.g. by Dabrowski (1987), where the presence of figures in communication between students was assumed to be one of the symptoms of understanding.

2 not to the numbers one.
Talking about mathematics and mathematical development of our students, it is useful to distinguish two types of figures of thought, -metaphors and metonymies. These two figures should be treated differently by a teacher or a text book writer. Metaphors are conscious, imagelike, based on analogy or similarity. Metonymies are subconscious shifts of reference without change of the semantic value. There are of course multitude of traces of such figures in our mathematical language in the form of idiomatic uses of some signs or groups of signs. A good example of a metonymy is the use of the equality sign "=" with the intended meaning "the result is". It is used with this meaning in many instances in school algebra, and often on pocket calculators. Due to lack of the assignment sign in school algebra, it is also used meaning the assignment operation, as in $x=1, y=0$. This is also called in school mathematics the substitution operation and the equality sign is often used with this "borrowed" signification. Taken literally, the phrase " $x=1$ " is pure nonsense. No variable could be equal to a number, and certainly ~~not to the numbers~~. The just mentioned metonymical usages of the equality sign are for those who see them for the first time in the process of learning. In many self evident circumstances such shifts of reference or "borrowings" are well established practice and often go without explanation, their meaning being subconsciously taken for granted. Explaining them might cause more trouble than clarification.

Knuth (1985) made interesting remarks concerning the lack of a generally accepted special sign in school algebra for the assignment operation. He however failed to see that it is usually being expressed by the metonymical use of "=". It would be advisable to use another sign in school algebra to this purpose, e.g. $x \leftarrow 1, y \leftarrow 0$. This sign is almost self-explaining, because of natural meaning of arrows in our culture. This sign was introduced in APL programming language of Iverson and it is still in use there. It is often chosen spontaneously and independently on many occasions in school, so that one may say that it is syntonic in the sense of Papert (1980).

Another typical figure used in school is the metonymy of letters for numbers. This is usually the first encounter of the notion of variable as it is used in school. On the formal level, it is much closer to the notion of an arbitrary constant than to the notion of variable as it is used in abstract algebra or some other formal mathematical disciplines. In primary arithmetics the metonymy of letters for numbers was successfully used by Davidov. I think that the power of metonymies he used gives some rationale for this approach.

Metaphors are useful in creating something new. New meanings, new signs, new shapes. Mathematical inventory is full of "petrified" metaphors, which have long been accepted into the standard "vocabulary" and have become accommodated idioms. One of such instances is the equality sign itself "=". Robert Recorde created it (1557) and, on this occasion, he wrote in his famous first manual of school algebra and arithmetics printed in English, "The Whetstone of Witte": "And to avoid the tedious repetition of these words «is equal to»

I will set as I do often in work use, a pair of parallels, or gemowe lines of one length, thus: =====, because no 2 things can be more equal. "It is clear from what he wrote that the equality sign was conceived by analogy. Two lines of equal length should remind one that nothing could be more equal. Of course this sign was a metaphor for Robert Recorde. It is no more so for us now. Today nobody knows any more which were the circumstances of its invention. Nor do we need to know it. For us it is a matter of pure convention.

There are more examples of metaphors and metonymies in mathematics. Some are described in this paper quoted above and there are more and more being created in every classroom. Most of them are local and they fade away as quickly as they come. Only some of them persist.

First remarks concerning figures of speech come from antiquity. The word "metaphor" comes from Aristotle, the word "metonymy" comes from Cicero. In his "Institutio Oratore" Quintilian wrote: "tropus est inquit verbi vel sermonis a propria significatione in aliam rem cum vertute mutatio", and elsewhere added "Transfertur ergo nomen aut, verbum ex eo loco, in quo proprium est, in eum, in quo aut proprium deest aut translatum proprio melius est" (vol. VIII, 6). We have now modern treatises on the subject like Le Guern (1972), Henry (1971) where the difference between metaphor and metonymy is especially well exposed and these monographs are very good preparation for the analysis of metaphors and metonymies in mathematics.

As in natural languages, in mathematics, every figure is deviation of a sort from what is considered to be the conventional usage, Cf. Bellert (1980/81), Black (1962, 1979), Reddy (1979) and many others. As a deviation and anomaly, it can always be considered a plain error. And from the point of view of a formalist teacher, it is an error. But it might not be a misconception. It may just be a symptom of a beginning of development in the right direction. It is usually all too easy to qualify it as an error, and nip it in the bud. Metaphors and metonymies were, throughout history, essential ingredients in the mathematical and scientific development, Kuhn (1979), Hao Wang (1986), Putnam (1986), and they are such for our students. They play greater role in creative learning, and perhaps a smaller and more hidden role in teaching ready made mathematics the formal way. It is remarkable that Green's (1979) article against metaphor could be interpreted as a praise of metonymy.

This last remark does not mean that any anomaly is a figure of thought and should be accepted as such. I only try to differentiate between those apparent errors which are intentional and are no errors at all and such anomalies that are true errors and need correction. Such anomalies that cannot be explained as figures of thought are either banal errors due to lack of suitable performance skill or serious errors due to the frame conflict, incompatibility, and real misconceptions. Some misconceptions are so deeply rooted in history and our own human nature that we call them epistemological obstacles, Sierpiska (1987).

It is especially important to identify errors due to misconception and frame conflict. In such situation more exercise, more drill and practice will not do and might be even detrimental to further learning. Frame conflicts are immune to facts, Schön (1979), Reddy (1979).

We should also be more attentive to the way the students interpret what we intend to be taken literally, no matter how careful, strict, and exact our words might appear to us. They might not appear so to other people with less mathematical experience. And what is even worse, such text might appear plain and literal but with a conveyed meaning different than intended.

Even if we do not like using figures of style or figures of thought too much, we should not assume that a literal text is always good remedy for ambiguities of message and its conveyed meaning. For instance, one such text which might be confusing is the usual definition of a function as we usually adopt it in mathematical school books. Our tests systematically show that when we use words only, and no pictures, then the conveyed meaning to the inexperienced reader is such as if there were only constant functions. No problem, you may say, then use pictures. Yes, we do, and almost all school books have them. They are used, because we know of the danger of confusion here. If no pictures are used, a rare event, then we explain in many ways, with long phrases what we mean by what was said, sometimes creating even more confusion. Yet our text, taken literally, says exactly what we wanted to say, nothing more and nothing less. The problems with the literal meaning of a text are not as rare in mathematical language used in school as it is usually assumed. For those interested in this problem I may only recommend the article of Rummelhart (1979) and Petrie (1979). I mention the verbal definition of a function only because it is so common. There are many other examples where what we say or write is done so with the belief that it is received literally, whereas it is often in fact interpreted differently.

The whetstone of witte,

whiche is the seconde parte of
Arithmetike: containing the extrac-
tion of roots: The Cosike practice,
with the rule of Equation: and
the woordes of Sundry
Numbers.

*Though many stones doe beate greates price,
The whetstone is for ever free
As needfull, and in woorkes as straunge:
Dulle thinges and hardes it will so change,
And make them sharpe, to right good use:
All artesmen knowe, they can not chuse,
But use his helpe, yet as men see,
Not sharpnessefemeth in it to bee.*

*The Grounde of artes did breede this stone:
Huse u greates, and make them one.
Hence if you list your wittes to whette,
Muche sharpnesse thereby shall you gette.
Dulle wittes hereby doe greatly mende,
Sharpe wittes are fined to their fulle ende.
Now proue, and praise, as you doe finde,
And to your selfe be not unkinde.*

These Bookes are to be sold, at
the Wolfe booke of Poules,
by Iohn Iaggons.

The Arte

as their woordes doe entende) to distinge it onely into
two parties. The first of the first is, When our numbers
equalle unto one other. And the second is, When our num-
ber is compared as equall unto 2 other numbers.

Alwaies willing you to remember, that you reduce
your numbers, to their least denominations, and
smallest soymes, before you proceed any farther.

And again, if your equation be soke, that the grea-
test denomination Cosike, be ioined to any parte of a
compound number, you shall tourne it so, that the
number of the greatest signe alone, make stand as
equall to the rest.

And this is all that needeth to be taughte, concern-
ing this woork.

Now to be, for ease alteration of equations. I will pre-
pound a few examples, because the extraction of these
roots, make the more aptly bee wrought. And to a-
void the tedious repetition of these woordes: is e-
quall to: I will sette as I doe often in woordes use, a
paire of paralleles, or Semiotick lines of one lengthe,
thus: ———, because no 2. thinges can be inoore
equall. And now make these numbers.

1. $14.22. + 15.9. = 71.9.$
 2. $20.22. = 13.9. = 102.9.$
 3. $26.3. + 10.22. = 9.3. + 10.22. = 212.9.$
 4. $19.22. + 192.9. = 103. + 1089. = 19.22.$
 5. $18.22. + 24.9. = 8.3. + 2.22.$
 6. $34.3. = 12.22. = 40.22. + 4809. = 9.3.$
1. In the first there appeareth. 2. numbers, that is
14.22.

I will set as I do often in work use, a pair of parallels, or gemowe lines of one length, thus: =====, because no 2 things can be more equal. "It is clear from what he wrote that the equality sign was conceived by analogy. Two lines of equal length should remind one that nothing could be more equal. Of course this sign was a metaphor for Robert Recorde. It is no more so for us now. Today nobody knows any more which were the circumstances of its invention. Nor do we need to know it. For us it is a matter of pure convention.

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The title page of "The Whetstone of Witte" of Robert Recorde and the page on which he introduced the equality sign as a pair of parallel lines of equal length, =====, "because no two thynges could be moare equalle". Note that the phrase "a pair of paralleles" could be interpreted as a synecdoche, they are not only parallel but of equal length as well, and the phrase "Gemowe lines of one length", - could be considered a metaphor. It is not usual to say "Gemowe" in place of "parallel". "Two things which are equal" might be a plain oxymoron for inexperienced readers in mathematics, and "two thynges that are moare equalle" might seem even more so.

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